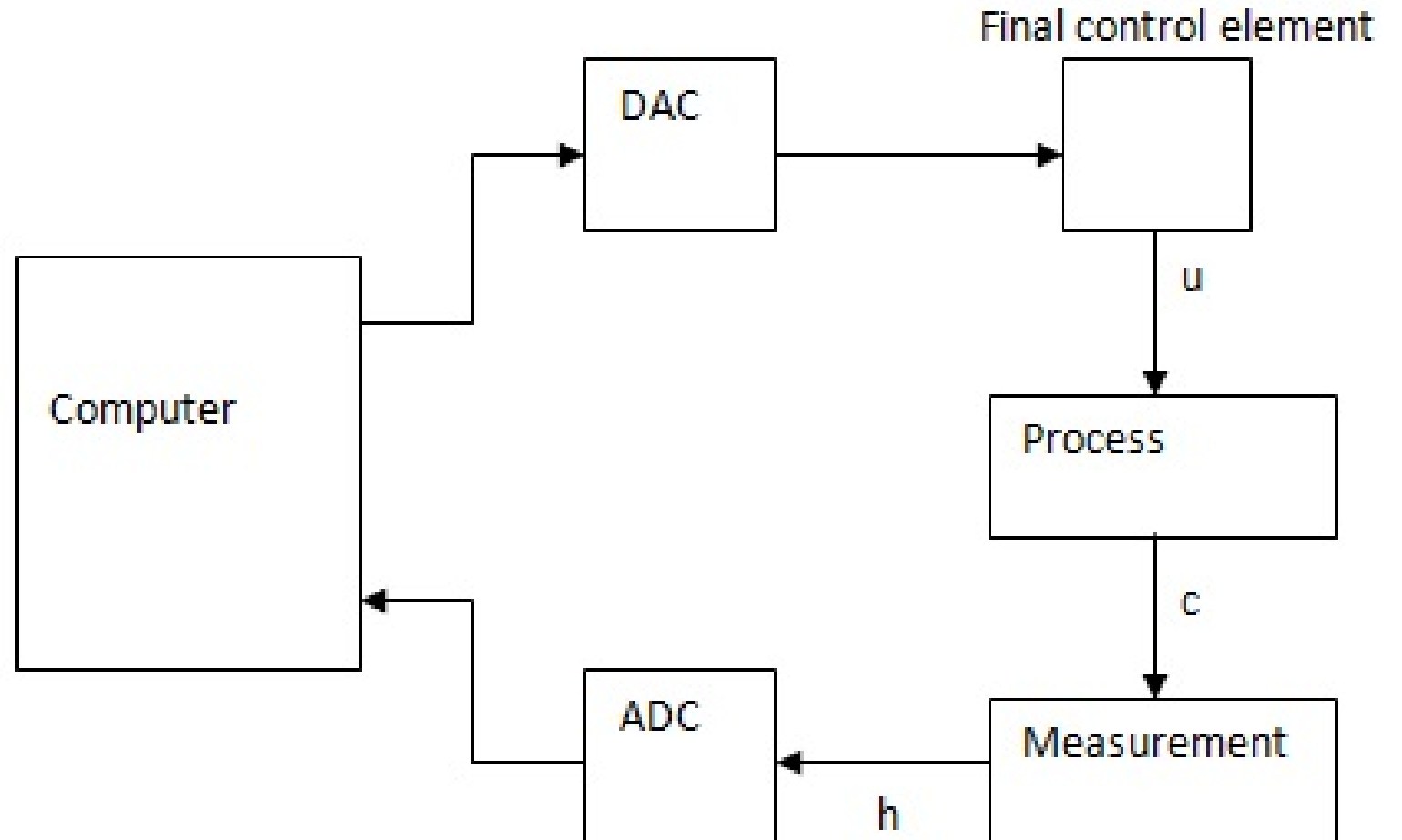


# Digital control

## Introduction part 3

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# Outline

- Overview of sensors used in computer control
- Mechanical system modelling
- Electrical system modelling
- Electromechanical systems modeling

# SENSORS USED IN COMPUTER CONTROL

- A sensor is a device that outputs a signal which is related to the measurement of (i.e. is a function of) a physical quantity such as:
- temperature, speed, force, pressure, displacement, acceleration, torque, flow, light or sound.
- Sensors are used in closed-loop systems in the feedback loops, and they provide information about the actual output of a plant.
- For example, a speed sensor gives a signal proportional to the speed of a motor and this signal is subtracted from the desired speed reference input in order to obtain the error signal.

# SENSORS USED IN COMPUTER CONTROL

- Sensors can be classified as **analog or digital**.
- Analog sensors are more widely available, and their outputs are analog voltages. For example, the output of an analog temperature sensor may be a voltage proportional to the measured temperature.
- Analog sensors can only be connected to a computer by using an A/D converter.
- Digital sensors are not very common and they have logic level outputs which can directly be connected to a computer input port.

# SENSORS USED IN COMPUTER CONTROL

- The choice of a sensor for a particular application depends on many factors such as the cost, reliability, required accuracy, resolution, range and linearity of the sensor.
- **Range.** The range of a sensor specifies the upper and lower limits of the measured variable for which a measurement can be made. For example, if the range of a temperature sensor is specified as 10–60 °C then the sensor should only be used to measure temperatures within that range.
- **Resolution.** The resolution of a sensor is specified as the largest change in measured value that will not result in a change in the sensor's output, i.e. the measured value can change by the amount quoted by the resolution before this change can be detected by the sensor.
- In general, the smaller this amount the better the sensor is, and sensors with a wide range have less resolution. For example, a temperature sensor with a resolution of 0.001K is better than a sensor with a resolution of 0.1 K.

# SENSORS USED IN COMPUTER CONTROL

- ***Repeatability***. The repeatability of a sensor is the variation of output values that can be expected when the sensor measures the same physical quantity several times.
- For example, if the voltage across a resistor is measured at the same time several times we may get slightly different results.
- ***Linearity***. An ideal sensor is expected to have a linear transfer function, i.e. the sensor output is expected to be exactly proportional to the measured value. However, in practice all sensors exhibit some amount of nonlinearity depending upon the manufacturing tolerances and the measurement conditions.
- ***Dynamic response***. The dynamic response of a sensor specifies the limits of the sensor characteristics when the sensor is subject to a sinusoidal frequency change. For example, the dynamic response of a microphone may be expressed in terms of the 3-dB bandwidth of its frequency response.

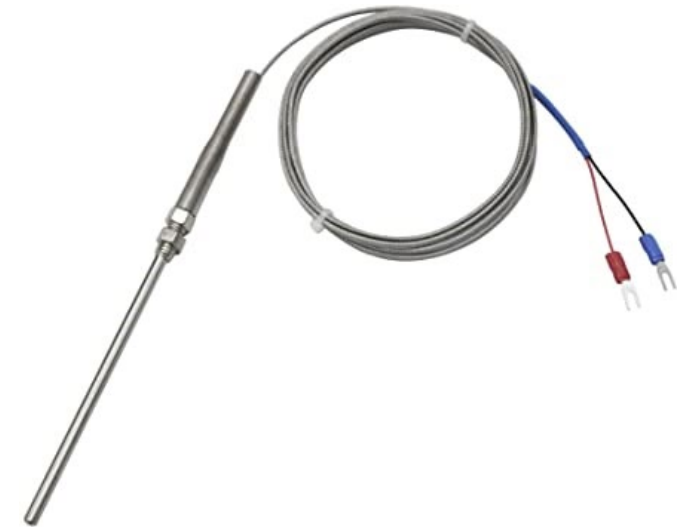
# Temperature sensors

**Table 1.1** Temperature sensors

Sensor	Temperature range (°C)	Accuracy ( $\pm$ °C)	Cost	Robustness
Thermocouple	-270 to +2600	1	Low	Very high
RTD	-200 to +600	0.2	Medium	High
Thermistor	-50 to +200	0.2	Low	Medium
Integrated circuit	-40 to +125	1	Low	Low

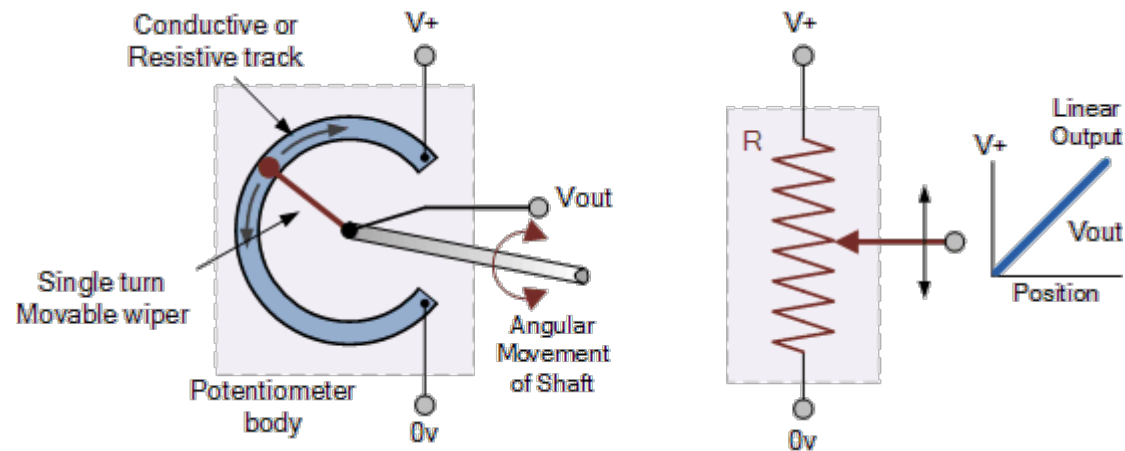
**Table 1.2** Temperature–data relationship of DS1620

Temperature (°C)	Digital output
+125	0 11111010
+25	0 00110010
0.5	0 00000001
0	0 00000000
-0.5	1 11111111
-25	1 11001110
-55	1 10010010



# Position sensors

- Position sensors are used to measure the position of moving objects.
- These sensors are basically of two types: sensors to measure linear movement, and sensors to measure angular movement.
- Examples: linear and rotary potentiometer





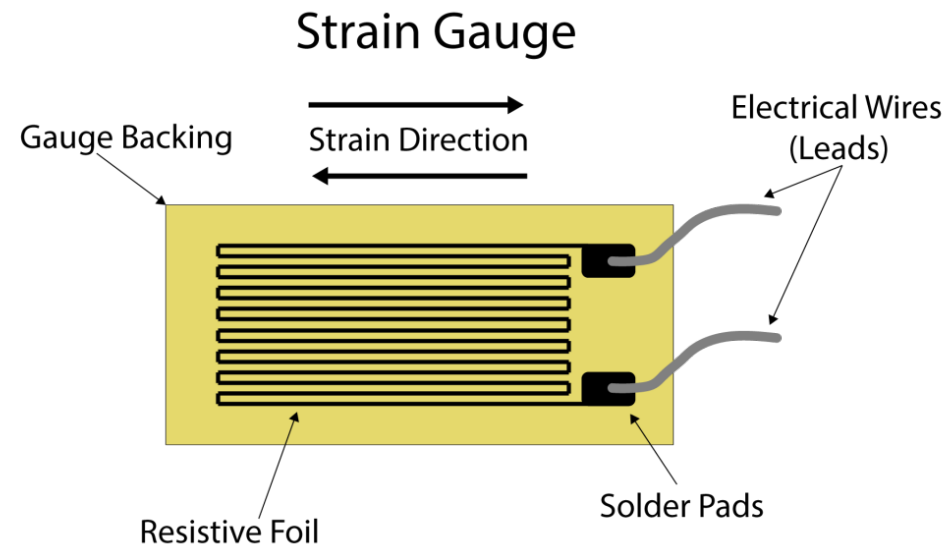
# Velocity and acceleration sensors

- Velocity is the differentiation of position, and in general position sensors can be used to measure velocity.
- The required differentiation can be done either in hardware (e.g. using operational amplifiers) or by the computer.
- For more accurate measurements velocity sensors should be used.
- There are two types of velocity sensors: linear sensors, and rotary sensors.



# Force sensors

- Force sensors can be constructed using position sensors. Alternatively, a strain gauge can be used to measure force accurately.
- There are many different types of strain gauges. A strain gauge can be made from capacitors and inductors, but the most widely used types are made from resistors.
- A wire strain gauge is made from a resistor, in the form of a metal foil.
- The principle of operation is that the resistance of a wire increases with increasing strain and decreases with decreasing strain.



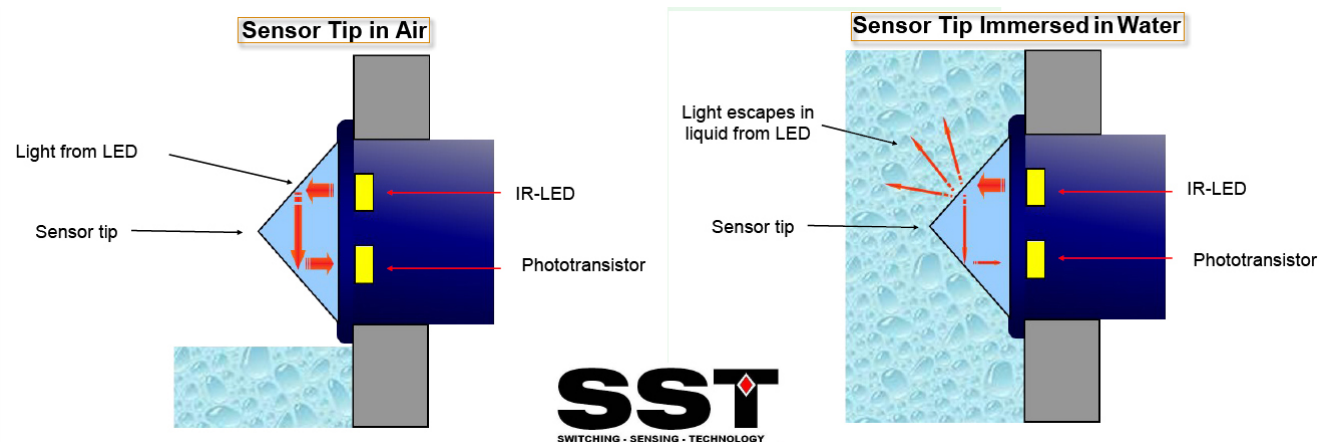
# Pressure sensors

- Early pressure measurement was based on using a flexible device (e.g. a diaphragm) as a sensor; the pressure changed as the device moved and caused a dial connected to the device to move and indicate the pressure.
- Nowadays, the movement is converted into an electrical signal which is proportional to the applied pressure. Strain gauges, capacitance change, inductance change, piezoelectric effect, optical pressure sensors and similar techniques are used to measure the pressure.



# Liquid sensors

- There are many different types of liquid sensors. These sensors are used to:
- detect the presence of liquid;
- measure the level of liquid;
- measure the flow rate of liquid, for example through a pipe.



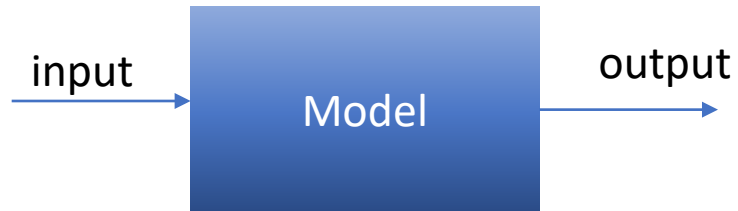
# Air flow sensors

- Air flow is usually measured using anemometers.
- A classical anemometer has a rotating vane, and the speed of rotation is proportional to the air flow.
- Hot wire anemometers have no moving parts. The sensor consists of an electrically heated platinum wire which is placed in the air flow.
- As the flow velocity increases the rate of heat flow from the heated wire to the flow stream increases and a cooling occurs on the electrode, causing its resistance to change.
- The flow rate is then determined from the change in the resistance.



# System modelling

- Modelling (British)/ Modeling (US) is the process of finding a mathematical model (relationship) between the inputs and the outputs of a given system such that for a given input, this relationship will give us the output of the system.



- The task of mathematical modelling is an important step in the analysis and design of control systems.
- The mathematical models of systems are obtained by applying the fundamental physical laws governing the nature of the components making these systems.
- For example, Newton's laws are used in the mathematical modelling of mechanical systems. Similarly, Kirchhoff's laws are used in the modelling and analysis of electrical systems.

# System modelling

- The mathematical model of a system is one or more differential equations describing the dynamic behaviour of the system.
- The Laplace transformation is applied to the mathematical model and then the model is converted into an algebraic equation which can be converted to a Z-domain or difference equation.
- The properties and behaviour of the system can then be represented as a block diagram, with the transfer function of each component describing the relationship between its input and output behaviour.

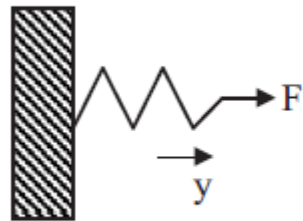
# Mechanical systems

- Models of mechanical systems are important in control engineering because a mechanical system may be a vehicle, a robot arm, a missile, or any other system which incorporates a mechanical component.
- Mechanical systems can be divided into two categories: translational systems and rotational systems.
- Some systems may be purely translational or rotational, whereas others may be hybrid, incorporating both translational and rotational components.

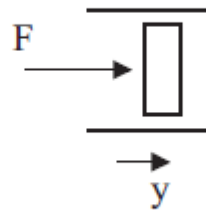


# Mechanical systems

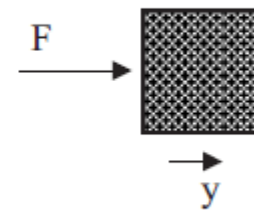
- The basic building blocks of translational mechanical systems are masses, springs, and dashpots
- The input to a translational mechanical system may be a force,  $F$ , and the output the displacement,  $y$ .
- *Springs* store energy and are used in most mechanical systems.



*Spring*



*Dashpot*

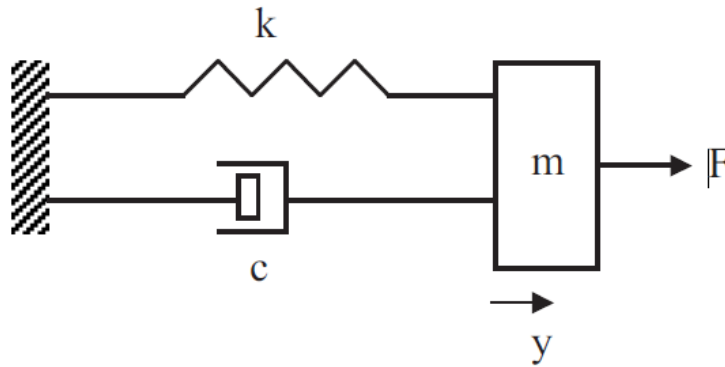


*Mass*

# Mechanical systems

## Example 2.1

Figure 2.3 shows a simple mechanical translational system with a mass, spring and dashpot. A force  $F$  is applied to the system. Derive a mathematical model for the system.



**Figure 2.3** Mechanical system with mass, spring and dashpot

# Mechanical systems

$$F - ky - c \frac{dy}{dt} = m \frac{d^2y}{dt^2} \quad (2.9)$$

or

$$F = m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky. \quad (2.10)$$

Equation (2.10) is usually written in the form

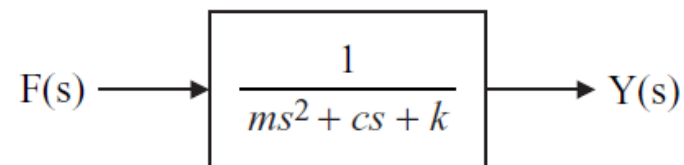
$$F = m\ddot{y} + c\dot{y} + ky. \quad (2.11)$$

Taking the Laplace transform of (2.11), we can derive the transfer function of the system as

$$F(s) = ms^2Y(s) + csY(s) + kY(s)$$

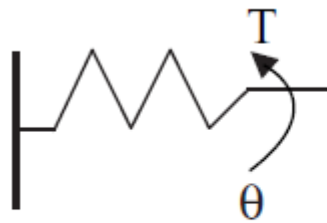
or

$$\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + cs + k}. \quad (2.12)$$

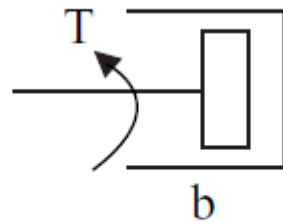


# Rotational Mechanical Systems

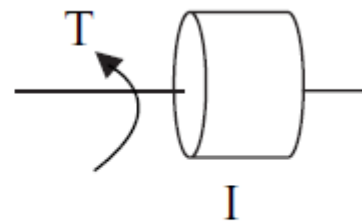
- The basic building blocks of rotational mechanical systems are the moment of inertia, torsion spring (or rotational spring) and rotary damper
- The input to a rotational mechanical system may be the torque,  $T$ , and the output the rotational displacement, or angle,  $\theta$ .



*Torsional spring*



*Rotational dashpot*



*Moment of inertia*

# Rotational Mechanical Systems

A *rotational spring* is similar to a translational spring, but here the spring is twisted. The relationship between the applied torque,  $T$ , and the angle  $\theta$  rotated by the spring is given by

$$T = k\theta, \quad (2.30)$$

The energy stored in a torsional spring when twisted by an angle  $\theta$  is given by

$$E = \frac{1}{2}k\theta^2. \quad (2.31)$$

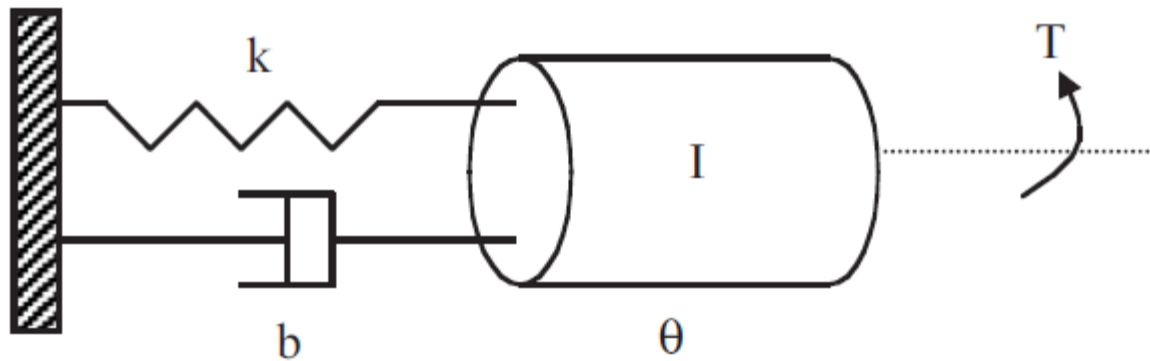
$$T = c\omega = c\frac{d\theta}{dt}.$$

$$T = Ia = I\frac{d\omega}{dt}$$

# Rotational Mechanical Systems

## Example 2.5

A disk of moment of inertia  $I$  is rotated (see Figure 2.9) with an applied torque of  $T$ . The disk is fixed at one end through an elastic shaft. Assuming that the shaft can be modelled with a rotational dashpot and a rotational spring, derive an equation for the mathematical model of this system.



**Figure 2.9** Rotational mechanical system

# Rotational Mechanical Systems

## *Solution*

The damper torque and spring torque oppose the applied torque. If  $\theta$  is the angular displacement from the equilibrium, we can write

$$T - b \frac{d\theta}{dt} - k\theta = I \frac{d^2\theta}{dt^2} \quad (2.36)$$

or

$$I \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} + k\theta = T. \quad (2.37)$$

Equation (2.37) is normally written in the form

$$I\ddot{\theta} + b\dot{\theta} + k\theta = T. \quad (2.38)$$

# Electrical systems



- The basic building blocks of electrical systems are the resistor, inductor and capacitor.
- The input to an electrical system may be the voltage,  $V$ , and current,  $i$ .
- The relationship between the voltage across a *resistor* and the current through it is given by

$$V_r = Ri,$$

- For an *inductor*, the potential difference across the inductor depends on the rate of change of current through the inductor, given by

$$v_L = L \frac{di}{dt},$$

- The relationship between the current through the capacitor and the voltage across it is given by

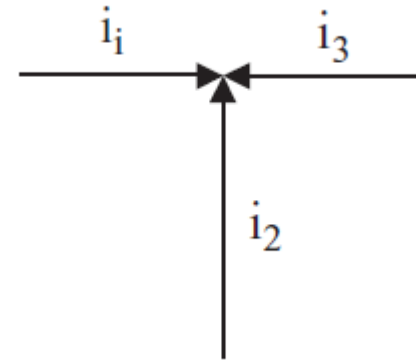
$$v_C = \frac{1}{C} \int i dt.$$



# Kirchhoff's current law

- The sum of the currents at a node in a circuit is zero, i.e. the total current flowing into any junction in a circuit is equal to the total current leaving the junction.

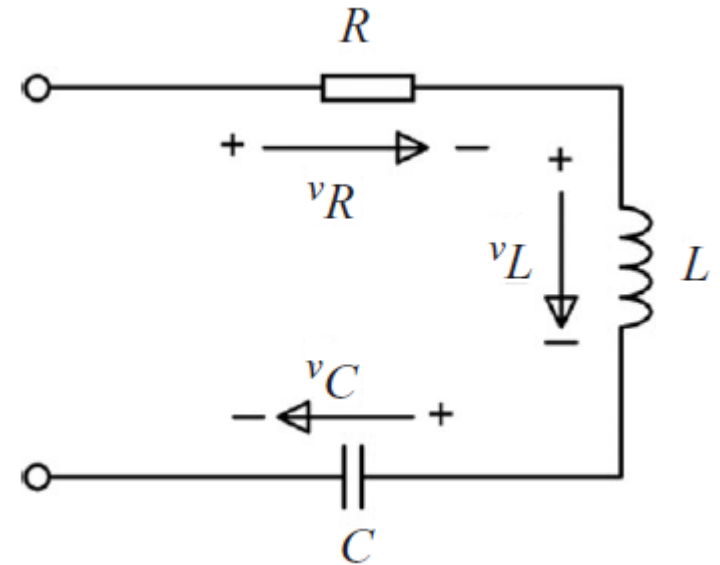
$$i_1 + i_2 + i_3 = 0$$



# Kirchhoff's voltage law

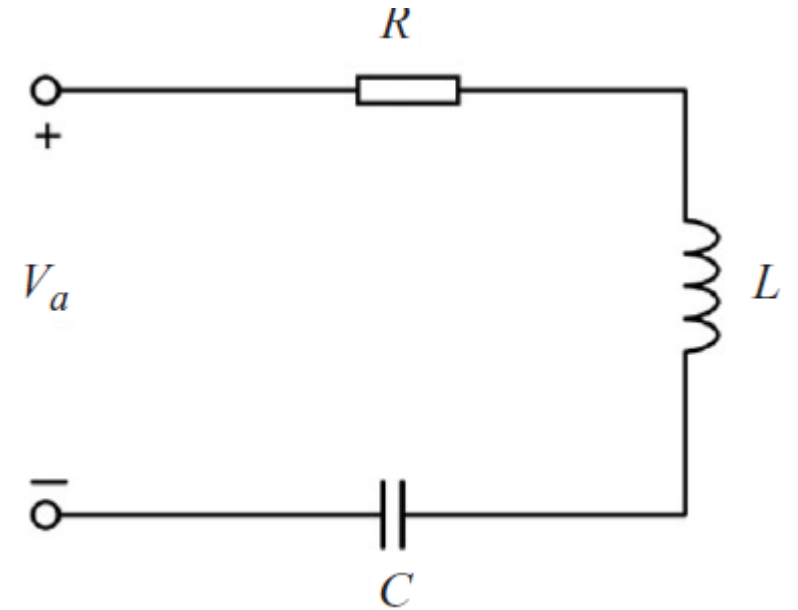
- The sum of voltages around any loop in a circuit is zero, i.e. in a circuit containing a source of electromotive force (e.m.f.), the algebraic sum of the potential drops across each circuit element is equal to the algebraic sum of the applied e.m.f.s.

$$v_R + v_L + v_C = 0.$$



# Example

- Figure shows a simple electrical circuit consisting of a resistor, an inductor and a capacitor.
- A voltage  $V_a$  is applied to the circuit.
- Derive an expression for the mathematical model for this system.



# Example

## *Solution*

Applying Kirchhoff's voltage law, we can write

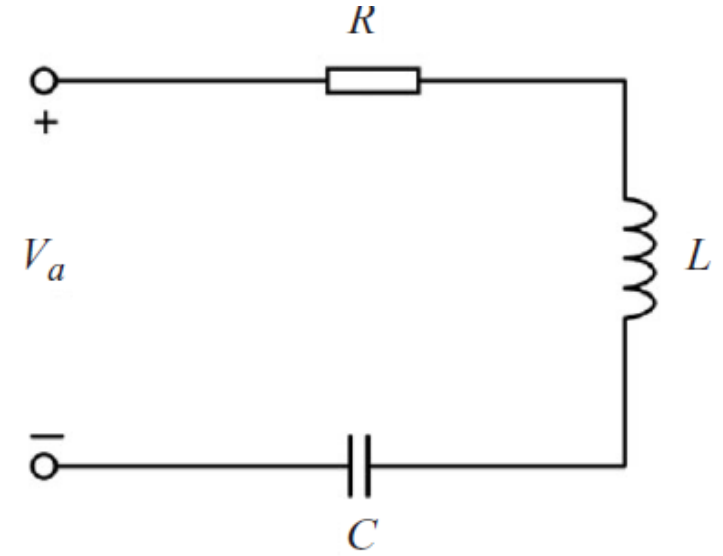
$$v_R + v_L + v_C = V_a$$

or

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt = V_a.$$

For the capacitor we can write

$$i = C \frac{dv_C}{dt}.$$



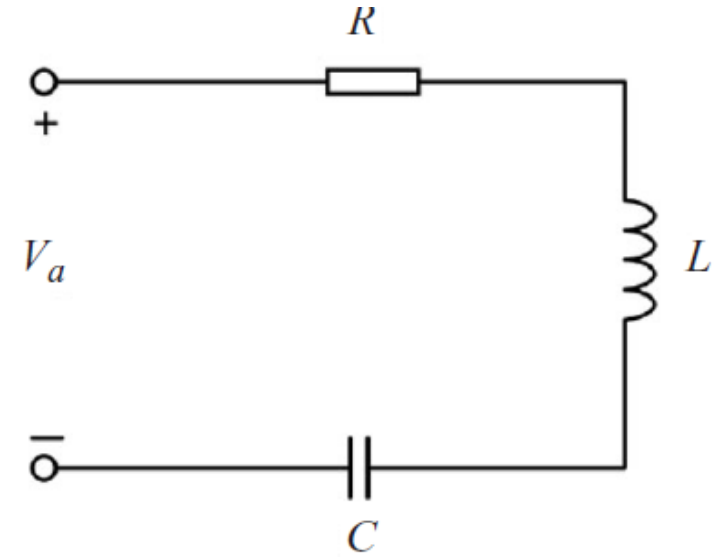
# Example

Substituting this into (2.67), we obtain

$$RC \frac{dv_C}{dt} + LC \frac{d^2v_C}{dt^2} + v_C = V_a$$

which can also be written as

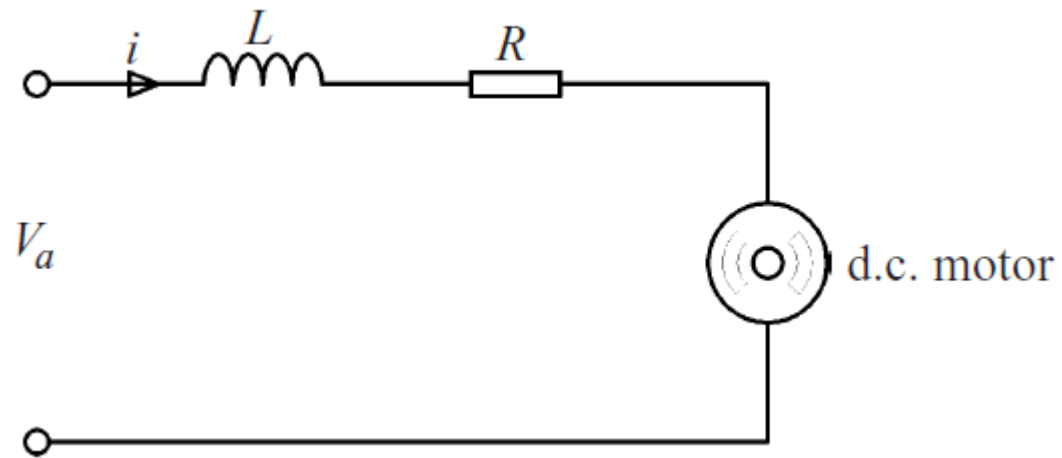
$$LC\ddot{v}_C + RC\dot{v}_C + v_C = V_a.$$



# ELECTROMECHANICAL SYSTEMS

- Devices convert electrical energy into mechanical movement - and sometimes vice versa.
- Most of the common electromechanical components, such as electric motors and solenoids are used in combination with mechanical parts to provide actuation or movement.
- Electromechanical systems such as electric motors and electric pumps are used in most industrial and commercial applications.

# Example: DC motor



1  $T = k_t i,$

2  $T = I \frac{d\omega}{dt}$

3  $I \frac{d\omega}{dt} = k_t i.$

# Example: DC motor

As the motor armature coil is rotating in a magnetic field there will be a *back e.m.f.* induced in the coil in such a way as to oppose the change producing it. This e.m.f. is proportional to the angular speed of the motor and is given by:

$$v_b = k_e \omega,$$

where  $v_b$  is the back e.m.f.,  $k_e$  is the back e.m.f. constant, and  $\omega$  is the angular speed of the motor.

Using Kirchhoff's voltage law, we can write the following equation for the motor circuit:

$$V_a - v_b = L \frac{di}{dt} + Ri, \quad (2.81)$$



# Example: DC motor

$$i = \frac{I}{k_t} \frac{d\omega}{dt}$$

$$\frac{LI}{k_t} \frac{d^2\omega}{dt^2} + \frac{RI}{k_t} \frac{d\omega}{dt} + k_e\omega = V_a.$$

... In many applications the motor inductance is small and can be neglected. The model then becomes

$$\frac{RI}{k_t} \frac{d\omega}{dt} + k_e\omega = V_a.$$

End